JEE Advanced 2026 Paper 2 - Mathematics Solutions

Section 1: Single Correct Option Type

Q.1

Let $|\vec{a}+\vec{b}| = \sqrt{21}$ and $|\vec{a}-\vec{b}| = 3$.
Since $\vec{a}$ and $(\vec{a}-\vec{b})$ are perpendicular, their dot product is zero:

$$\vec{a}\cdot(\vec{a}-\vec{b}) = 0 \implies |\vec{a}|^2 = \vec{a}\cdot\vec{b}$$

Squaring the given magnitudes:

$$|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 21+9 = 30 \implies 2(|\vec{a}|^2+|\vec{b}|^2) = 30 \implies |\vec{a}|^2+|\vec{b}|^2 = 15$$

$$|\vec{a}+\vec{b}|^2 - |\vec{a}-\vec{b}|^2 = 21-9 = 12 \implies 4\vec{a}\cdot\vec{b} = 12 \implies \vec{a}\cdot\vec{b} = 3$$

Substitute $\vec{a}\cdot\vec{b} = 3$ into our first equation:

$$|\vec{a}|^2 = 3 \implies |\vec{b}|^2 = 12$$

The area of triangle OPR, with vertices at $\vec{0}$, $\vec{a}$, and $\vec{a}+\vec{b}$, is:

$$\text{Area} = \frac{1}{2}|\vec{a}\times(\vec{a}+\vec{b})| = \frac{1}{2}|\vec{a}\times\vec{b}|$$

Using the identity $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$:

$$|\vec{a}\times\vec{b}|^2 = (3)(12) - (3)^2 = 36-9 = 27 \implies |\vec{a}\times\vec{b}| = 3\sqrt{3}$$

$$\text{Area} = \frac{3\sqrt{3}}{2}$$

Correct Option: (C)

Q.2

The parabola is $y^2 = 16x$, so $a = 4$.
The tangent T at $(64, 32)$ is:

$$32y = 8(x+64) \implies y = \frac{1}{4}x + 16$$

The slope of T is $m_T = 1/4$. Since L is perpendicular to T, its slope is $m_L = -4$.
The equation of a tangent to $y^2 = 4ax$ with slope m is $y = mx + \frac{a}{m}$. The point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.
For $m = -4$:

$$(x_1, y_1) = \left(\frac{4}{16}, \frac{8}{-4}\right) = \left(\frac{1}{4}, -2\right)$$

The focus of the parabola is $(a, 0) = (4, 0)$. The distance between $\left(\frac{1}{4}, -2\right)$ and $(4, 0)$ is:

$$\text{Distance} = \sqrt{\left(4-\frac{1}{4}\right)^2 + (0-(-2))^2} = \sqrt{\left(\frac{15}{4}\right)^2 + 4} = \sqrt{\frac{225}{16}+4} = \sqrt{\frac{289}{16}} = \frac{17}{4}$$

Correct Option: (C)

Q.3

Rearranging the differential equation:

$$\frac{dy}{dx} = \frac{y^3(e^{5x}+1)}{e^x(1+y^4)}$$

Separate the variables and integrate:

$$\int \left(\frac{1}{y^3} + y\right) dy = \int (e^{4x} + e^{-x}) dx$$

$$-\frac{1}{2y^2} + \frac{y^2}{2} = \frac{e^{4x}}{4} - e^{-x} + C$$

Use the initial condition $y(0) = \frac{1}{\sqrt{2}}$:

$$-\frac{1}{2(1/2)} + \frac{1/2}{2} = \frac{1}{4} - 1 + C \implies -1 + \frac{1}{4} = -\frac{3}{4} + C \implies C = 0$$

Now, evaluate at $x = \log_e 2$ (so $e^x = 2$):

$$\frac{y^2}{2} - \frac{1}{2y^2} = \frac{2^4}{4} - 2^{-1} = 4 - \frac{1}{2} = \frac{7}{2}$$

$$y^4 - 7y^2 - 1 = 0 \implies y^2 = \frac{7+\sqrt{53}}{2} \quad \text{(Taking the positive root since } y^2 > 0\text{)}$$

$$y = \sqrt{\frac{7+\sqrt{53}}{2}}$$

Correct Option: (B)

Q.4

$$I = \int_0^2 \frac{1}{3^x+3}\, dx$$

Substitute $u = 3^x$, then $du = 3^x \ln 3\, dx = u \ln 3\, dx \implies dx = \frac{du}{u \ln 3}$.
The limits change from $x \in [0, 2]$ to $u \in [1, 9]$.

$$I = \int_1^9 \frac{1}{u+3} \frac{du}{u \ln 3} = \frac{1}{\ln 3} \int_1^9 \frac{1}{u(u+3)}\, du$$

Using partial fractions:

$$I = \frac{1}{3\ln 3} \int_1^9 \left(\frac{1}{u} - \frac{1}{u+3}\right) du = \frac{1}{3\ln 3} [\ln u - \ln(u+3)]_1^9$$

$$I = \frac{1}{3\ln 3}((\ln 9 - \ln 12) - (\ln 1 - \ln 4)) = \frac{1}{3\ln 3} \ln\left(\frac{9}{12} \times 4\right) = \frac{\ln 3}{3\ln 3} = \frac{1}{3}$$

Correct Option: (B)

Section 2: Multiple Correct Options Type

Q.5
The function is $f(x) = \frac{d^{10}}{dx^{10}}((x^2-1)^{10})$.
Expanding $(x^2-1)^{10} = x^{20} - 10x^{18} + 45x^{16} - \dots - 252x^{10} + \dots$
Taking the 10^{th} derivative:

$$f(x) = \frac{20!}{10!}x^{10} - 10\frac{18!}{8!}x^8 + \dots - 252(10!)$$

- **(A)** The coefficient of x^8 is $-10\left(\frac{18!}{8!}\right)$. **(TRUE)**
- **(B)** $f(1) = 10!2^{10}$ (by Leibniz rule) and since $f(x)$ is even, $f(1) + f(-1) = 10!2^{11}$. **(TRUE)**
- **(C)** The highest power is x^{10}, so the degree is 10. **(TRUE)**
- **(D)** The constant term is $-252 \times 10!$, which does not equal $-\frac{10!}{5!}$. **(FALSE)**

Correct Options: (A), (B), (C)

Q.6
For $ax^2+bx+c=0$ to have integer solutions with a, b, c as positive integers in A.P., roots must be negative integers α, β.
From the A.P. condition, $2b = a + c \implies -2a(\alpha+\beta) = a + a\alpha\beta \implies (\alpha+2)(\beta+2) = 3$.
Solving this gives $\alpha = -3$ and $\beta = -5$ as the only valid integer roots.
Thus, sum of roots is $-8 \implies b = 8a$, and product is $15 \implies c = 15a$.

- **(A)** $c - b = 15a - 8a = 7a$, a multiple of a. **(TRUE)**
- **(B)** Roots are $-3, -5$, both odd. **(TRUE)**
- **(C)** If $c = 15 \implies a = 1$. Then $b = 8$, so $ab = 8$. **(TRUE)**
- **(D)** $x = 3$ is not a root. **(FALSE)**

Correct Options: (A), (B), (C)

Q.7
Line L passing through $P(1,2,-1)$ and $Q(2,3,1)$ has direction ratio $(1,1,2)$.
Coordinate of any point on L is $(1+\lambda, 2+\lambda, -1+2\lambda)$. Let this be S.
$\vec{RS}$ is perpendicular to L, so $1(1+\lambda-4) + 1(2+\lambda+1) + 2(-1+2\lambda-5) = 0 \implies \lambda = 2$.
Thus, $S = (3,4,3)$ and $RS = \sqrt{(-1)^2 + 5^2 + (-2)^2} = \sqrt{30}$.
Using the section formula for $|PS| : |ST| = 1 : 2$ internally, we find $T = (7,8,11)$.

- **Area:** Base $PT = 3 \cdot PS = 6\sqrt{6}$. Area $= \frac{1}{2} \cdot 6\sqrt{6} \cdot \sqrt{30} = 18\sqrt{5}$. **(D) is TRUE, (C) is FALSE.**
- **Orthocentre:** Geometric intersections of altitudes yield $\left(\frac{23}{5}, -4, \frac{31}{5}\right)$. **(A) is TRUE, (B) is FALSE.**

Correct Options: (A), (D)

Q.8
Linear Differential Equation: $\frac{dy}{dx} - \frac{y}{x} = -x^2 \implies$ Integrating factor $= \frac{1}{x}$.

$$y\left(\frac{1}{x}\right) = \int -x\,dx = -\frac{x^2}{2} + C \implies y = f(x) = -\frac{x^3}{2} + Cx$$

Using $y(1) = 0 \implies C = \frac{1}{2}$, so $f(x) = \frac{x-x^3}{2}$.
$f'(x) = \frac{1-3x^2}{2}$. For $x > 0$, $f'(x) = 0$ at $x = \frac{1}{\sqrt{3}}$ and $f''(x) = -3x < 0$, yielding a local maximum.
(B) is TRUE, (A) is FALSE.
$f'(x)$ is negative in $(1, 2)$, so it is decreasing. **(C) is FALSE.**
Intersection: $\frac{x-x^3}{2} = 4x^3 - 5x^2 + \frac{3}{2}x \implies 9x^3 - 10x^2 + 2x = 0$. Since $x > 0$, $9x^2 - 10x + 2 = 0$ yields 2 distinct positive roots. **(D) is TRUE.**
Correct Options: (B), (D)

Q.9
Calculating $ST = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$. So $a = 0, b = -1, c = 1, d = 1$.

- **(A)** $\frac{-1+0}{1+i} = -\frac{1-i}{2} \neq i$. **(FALSE)**
- **(B)** $\frac{0-1}{\omega+1} = \frac{-1}{-\omega^2} = \omega$. **(TRUE)**
- **(C)** Condition does not strictly mandate m is a multiple of 8. **(FALSE)**
- **(D)** For $z = x + iy \in H (y > 0)$, $\frac{-1}{z+1} = \frac{-(x+1)+iy}{(x+1)^2+y^2}$, which has a positive imaginary part, remaining in H. **(TRUE)**

Correct Options: (B), (D)

Section 3: Numerical Value Type

Q.10
$g(f(x)) = 2^x$ for $x \in A$. Outputs are $2, 4, 8, 16, 32$. All 5 outputs are distinct, so f must be an injective mapping from A to B.
Total injective mappings without restrictions $= {}^7P_5 = 2520$.
Let C_2 be arrangements where $f(2) = 2$ (${}^6P_4 = 360$) and C_4 be arrangements where $f(4) = 4$ (${}^6P_4 = 360$).
Arrangements where both occur: $C_{2\cap 4} = {}^5P_3 = 60$.
Using the principle of inclusion-exclusion, valid mappings $= 2520 - (360 + 360) + 60 = 1860$.
Answer: 1860

Q.11
Total possibilities for selection $= \binom{11}{6} = 462$.
The pair of selected books (Math, Physics) and their absolute differences X can be:

- $(6, 0) \to X = 6$, count $= 1$
- $(5, 1) \to X = 4$, count $= 30$
- $(4, 2) \to X = 2$, count $= 150$
- $(3, 3) \to X = 0$, count $= 200$
- $(2, 4) \to X = 2$, count $= 75$
- $(1, 5) \to X = 4$, count $= 6$

Mean $a = \frac{1}{462}[6(1) + 4(36) + 2(225) + 0(200)] = \frac{600}{462} = \frac{100}{77}$.
Therefore, $77a = 100$.
Answer: 100

Q.12

Using summation identities:

$$\sum_{i=1}^{10} x_i = 50 \quad \text{and} \quad \sum_{i=1}^{10} x_i^2 = 10(\text{Var} + \text{Mean}^2) = 10(7+25) = 320$$

For the first 8: $\sum_{i=1}^{8} x_i = 32$ and $\sum_{i=1}^{8} x_i^2 = 8(3.5+16) = 156$.
Thus, $x_9 + x_{10} = 18$ and $x_9^2 + x_{10}^2 = 164$.

$$(x_9 + x_{10})^2 = 164 + 2x_9x_{10} \implies 324 = 164 + 2x_9x_{10} \implies x_9x_{10} = 80$$

Roots for $t^2 - 18t + 80 = 0$ are $8, 10$. Since $x_9 < x_{10} \implies x_9 = 8, x_{10} = 10$.
$3(8) + 2(10) = 44$.
Answer: 44

Q.13

Ellipse E: $e_E = \sqrt{1 - \frac{12}{18}} = \frac{1}{\sqrt{3}}$. Foci are $(\pm\sqrt{6}, 0)$.
Hyperbola H has $e_H = \sqrt{3}$ and shares foci, meaning $a_He_H = \sqrt{6} \implies a_H = \sqrt{2}$. $b_H^2 = 2(3-1) = 4$.
H equation: $2x^2 - y^2 = 4$.
Intersection with parabola $x^2 = \sqrt{5}y$: $2\sqrt{5}y - y^2 = 4 \implies y = \sqrt{5} \pm 1$.
Points in quadrant 1 are $(\sqrt{5-\sqrt{5}}, \sqrt{5}-1)$ and $(\sqrt{5+\sqrt{5}}, \sqrt{5}+1)$.
Distance $d^2 = 14 - 4\sqrt{5} \implies a = 14, b = -4$.
$a - b = 18$.
Answer: 18

Q.14

For $f(x)$, $\sin^2(\pi(x - [x]))$ vanishes precisely when x is an integer. It jumps when x^3 changes integers (except at the 5 integer values of x). Between $(-27, 27)$, there are 53 integers. $|A| = 53 - 5 = 48$.
For $g(x)$, discontinuities occur where $x - [x]$ jumps. Except at $x = 0$, $g(x)$ is discontinuous at $\{-2, -1, 1, 2\}$. $|B| = 4$.
Since $A \cap B = \emptyset \implies |A \cap B| = 0$.
$|A| + 2|B| - |A \cap B| = 48 + 8 = 56$.
Answer: 56

Section 4: Question Stem Numerical Types

Q.15

Equating curves C_1 and C_2:

$$e^{-x} = e^{-x}(\sin x + \cos x) \implies \sin x + \cos x = 1 \implies \sin(x + \pi/4) = 1/\sqrt{2}$$

For $x \in [0, 10\pi]$, $x = 2k\pi$ and $x = 2k\pi + \pi/2$. Over 5 full cycles (k=0 to 4), this gives 10 points, plus the endpoint 10π.
Total points $n = 11$.
Answer: 11

Q.16

Integrate the absolute difference over the roots: $\alpha_1 = 0$, $\alpha_4 = 5\pi/2$.

$$\beta = \int_0^{5\pi/2} |e^{-x} - e^{-x}(\sin x + \cos x)|\, dx$$

Splitting across the ranges where the function crosses zero yields:

$$\beta = 2e^{-\pi/2} + e^{-5\pi/2}$$

We evaluate:

$$-\frac{1}{\pi}\log_e(\beta - 2e^{-\pi/2}) = -\frac{1}{\pi}\log_e(e^{-5\pi/2}) = \frac{5\pi/2}{\pi} = 2.5$$

Answer: 2.50

Q.17

Solving ellipses $x^2+4y^2 = 1$ and $4x^2+y^2 = 1$ simultaneously gives $x = y$ and $P = (1/\sqrt{5}, 1/\sqrt{5})$.
The tangent slopes m_1 and m_2 at P are $-1/4$ and -4, respectively.
The angle $\tan\theta = \left|\frac{-1/4-(-4)}{1+1}\right| = \frac{15}{8}$.
$4\tan\theta = 4(15/8) = 7.5$.
Answer: 7.50

Q.18

By symmetric geometry, the common internal area α is identical across the 4 quadrants.

$$\alpha = 4 \times \left(\frac{1}{2}\arctan(2)\right) = 2\arctan(2)$$

$$\cot\alpha = \cot(2\arctan 2) = \frac{1-\tan^2(\arctan 2)}{2\tan(\arctan 2)} = \frac{1-4}{2(2)} = -\frac{3}{4} = -0.75$$

Answer: -0.75