JEE Advanced 2026 Paper 1 - Mathematics Solutions

SECTION 1: Single Correct Type

Q.1: Consider the function $f(x) = \sqrt{x}\ln(x) - x + 1$ for $x \in (0, \infty)$.
Let's find the first and second derivatives of the function.

$$f'(x) = \frac{1}{2\sqrt{x}}\ln(x) + \sqrt{x}\left(\frac{1}{x}\right) - 1 = \frac{\ln(x) + 2}{2\sqrt{x}} - 1$$

$$f''(x) = \frac{\frac{1}{x}(2\sqrt{x}) - (\ln(x) + 2)\frac{1}{\sqrt{x}}}{4x} = \frac{-\ln(x)}{4x\sqrt{x}}$$

For $x \in (0, 1)$, $\ln(x) < 0$, which means $f''(x) > 0$. Thus, $f'(x)$ is strictly increasing in $(0, 1)$, making statement (A) false.

To find local extrema, we set $f'(x) = 0$:

$$\frac{\ln(x) + 2}{2\sqrt{x}} = 1 \implies \ln(x) + 2 = 2\sqrt{x}$$

Let $g(x) = 2\sqrt{x} - \ln(x) - 2$. Taking its derivative: $g'(x) = \frac{1}{\sqrt{x}} - \frac{1}{x} = \frac{\sqrt{x}-1}{x}$.

Setting $g'(x) = 0$ gives $x = 1$. The minimum value of $g(x)$ occurs at $x = 1$, where $g(1) = 2(1)-0-2 = 0$. Since the absolute minimum of $g(x)$ is zero, $g(x) \geq 0$ for all $x > 0$.

This implies $2\sqrt{x} \geq \ln(x) + 2$, and consequently $f'(x) \leq 0$ for all $x \in (0, \infty)$. Because the derivative is always non-positive and only zero at a single point ($x = 1$), $f(x)$ is a strictly decreasing function. Therefore, it has neither a local maximum nor a local minimum.
Correct Option: (D)

Q.2: Finding the radius of the circle passing through points P, Q, and R.

1. **Point P:** Lies on $y = x^2$ with tangent slope 4. $y' = 2x = 4 \implies x = 2, y = 4$. So, $P = (2, 4)$.
2. **Point Q:** Lies in the 1st quadrant on $x^2 + y^2 = 2$ with tangent slope -1. Differentiating gives $-\frac{x}{y} = -1 \implies x = y$. Substituting into the circle equation yields $2x^2 = 2 \implies x = 1$. So, $Q = (1, 1)$.
3. **Point R:** Lies in the 1st quadrant on $x^2 + 4y^2 = 8$ with tangent slope -1/2. Differentiating gives $-\frac{x}{4y} = -\frac{1}{2} \implies x = 2y$. Substituting yields $(2y)^2 + 4y^2 = 8 \implies 8y^2 = 8 \implies y = 1$. Thus, $x = 2$. So, $R = (2, 1)$.

Plotting these coordinates, $P(2, 4)$ and $R(2, 1)$ share an x-coordinate, forming a vertical line. $Q(1, 1)$ and $R(2, 1)$ share a y-coordinate, forming a horizontal line. Thus, $\angle QRP = 90°$.

Because $\triangle PQR$ is a right-angled triangle at R, the circumcircle's diameter is the hypotenuse PQ.

$$\text{Diameter} = \sqrt{(2-1)^2 + (4-1)^2} = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\text{Radius} = \frac{\sqrt{10}}{2} = \sqrt{\frac{10}{4}} = \sqrt{\frac{5}{2}}$$

Correct Option: (C)

Q.3: Matrices obtained by elementary row transformations.
A matrix obtained by performing elementary row transformations on the identity matrix must be invertible (its determinant cannot be zero). We calculate the determinants of the given options:

- (A) All rows are identical $\implies$ det $= 0$.
- (B) R_3 is a linear combination of other rows. If you do $R_2 - 2R_1 \to [0, 1, 2]$ and $R_3 - 2R_1 \to [0, 3, 6]$, we see R_3 is a multiple of the modified $R_2 \implies$ det $= 0$.
- (C) Expanding the determinant: $1(3-8) - 1(2-4) + 1(4-3) = -5 + 2 + 1 = -2 \neq 0$.
- (D) $R_1 + R_2 = [0, 2, 3]$, which perfectly matches $R_3 \implies$ det $= 0$.

Since only matrix (C) is invertible, it is the only one that can be derived from the identity matrix via elementary transformations.
Correct Option: (C)

Q.4: Principal values of inverse trigonometric functions.
Let's evaluate the expression $E = \cot^{-1}(\cot(-11)) + 10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 10\sin(2\tan^{-1}(2))$ term by term:

1. **Term 1:** The range of $\cot^{-1}$ is $(0, \pi)$. We need an angle $\theta \in (0, \pi)$ such that $\cot(\theta) = \cot(-11)$. Since the period of cotangent is π, $\theta = -11 + 4\pi \approx 1.56$, which safely falls in $(0, 3.14)$. So, this term evaluates to $4\pi - 11$.
2. **Term 2:** $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$. Thus, $10\sin\left(2 \cdot \frac{\pi}{4}\right) = 10\sin\left(\frac{\pi}{2}\right) = 10$.
3. **Term 3:** Let $\alpha = \tan^{-1}(2)$, so $\tan(\alpha) = 2$. We need $10\sin(2\alpha)$. Using the identity $\sin(2\alpha) = \frac{2\tan\alpha}{1+\tan^2\alpha}$, we get $10\left(\frac{2(2)}{1+2^2}\right) = 10\left(\frac{4}{5}\right) = 8$.

Summing them up: $(4\pi - 11) + 10 + 8 = 4\pi + 7$.
Correct Option: (C)

SECTION 2: Multiple Correct Type

Q.5: Box I and Box II probability.
Total balls: Box I (6 Red, 9 Green) = 15. Box II (8 Red, 12 Green) = 20. Total combined = 35. Total Red = 14, Total Green = 21.

$P(E_1) = \frac{15}{35} = \frac{3}{7}$, $P(E_2) = \frac{20}{35} = \frac{4}{7}$.
$P(F_1) = \frac{14}{35} = \frac{2}{5}$, $P(F_2) = \frac{21}{35} = \frac{3}{5}$.

- **(A)** $P(E_1 \cap F_1)$ is drawing a red ball specifically from Box I, which is $\frac{6}{35}$. $P(E_1)P(F_1) = \left(\frac{3}{7}\right)\left(\frac{2}{5}\right) = \frac{6}{35}$. Since they match, they are independent. (TRUE)
- **(B)** Since E_1, F_1 are independent partitions, their complements E_2, F_2 are also independent. (FALSE)
- **(C)** $P(F_1|E_1) = \frac{6}{15} = \frac{2}{5}$. $P(F_1|E_2) = \frac{8}{20} = \frac{2}{5}$. They are equal. (TRUE)
- **(D)** $P(F_2|E_2) = \frac{12}{20} = \frac{3}{5}$. The condition states $\frac{2}{5} > \frac{3}{5}$, which is false. (FALSE)

Correct Option(s): (A), (C)

Q.6: Plane geometry.

Line L has direction $\vec{d} = \langle 2, 3, 1 \rangle$ and passes through $(1, 3, -2)$.
Plane P_0 normal is $\vec{n}_0 = \langle 1, 2, 3 \rangle$.
The normal to Plane P is $\vec{n} = \vec{d} \times \vec{n}_0 = \langle (9-2), -(6-1), (4-3) \rangle = \langle 7, -5, 1 \rangle$.
Equation of P: $7(x-1) - 5(y-3) + 1(z+2) = 0 \implies 7x - 5y + z = -10$.

- **(A)** Matches our derived equation. (TRUE)
- **(B)** Plane P_1 is parallel to P and passes through $(4, 2, 2)$, yielding $7(4) - 5(2) + 2 = 20$. The equation is $7x - 5y + z = 20$. Distance is $\frac{|20-(-10)|}{\sqrt{75}} = \frac{30}{5\sqrt{3}} = 2\sqrt{3}$. Option says 30. (FALSE)
- **(C)** Distance of P from origin is $\frac{|-10|}{\sqrt{75}} = \frac{2}{\sqrt{3}}$. Option says $2\sqrt{3}$. (FALSE)
- **(D)** Acute angle between P and $x + 2y + z = 3$ (Normal $\langle 1, 2, 1 \rangle$): $\cos\theta = \frac{|7(1)-5(2)+1(1)|}{\sqrt{75}\sqrt{6}} = \frac{2}{15\sqrt{2}} = \frac{\sqrt{2}}{15}$. Option states something else. (FALSE)

Correct Option(s): (A)

Q.7: Continuous and Differentiable Functions, $g(x) = xf(x)$.

- **(A)** If $f(x) = 1/x$ for $x \neq 0$ and $f(0) = 0$, $g(x)$ becomes 1 for $x \neq 0$ and 0 at $x = 0$, which isn't continuous. (FALSE)
- **(B)** $g'(0) = \lim_{h\to 0} \frac{hf(h)-0}{h} = \lim_{h\to 0} f(h)$. If f is continuous at 0, this limit equals $f(0)$, so g is differentiable. (TRUE)
- **(C)** If g is differentiable at 0, $\lim_{h\to 0} f(h)$ exists (let's call it L). But L doesn't have to equal $f(0)$. (FALSE)
- **(D)** As shown above, the existence of $g'(0)$ inherently requires $\lim_{x\to 0} f(x)$ to exist. (TRUE)

Correct Option(s): (B), (D)

Q.8: Matrices sequences.

$M = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$. The characteristic equation is $(\lambda - 1)^2 = 0$. $M = I + N$ where $N = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$. Note

that $N^2 = 0$.
Thus, $M^k = (I+N)^k = I + kN = \begin{pmatrix} 1+k & -k \\ k & 1-k \end{pmatrix}$.

- **(A)** Since M has a repeated eigenvalue of 1 and $M \neq I$, its Jordan canonical form is $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. (TRUE)
- **(B)** $\sum_{k=1}^{26} M^k = 26I + (\sum_{k=1}^{26} k)N = 26I + 351N$. Entry a (top left) is $26 + 351(1) = 377$. Option states 378. (FALSE)
- **(C)** Because $\det(M) = 1$, $\det(M^{26}) = 1$. An integer matrix with determinant 1 has an inverse with integer entries. (TRUE)
- **(D)** The system matrix is $S + tI$, where $S = 26I + 351N$. The eigenvalues of N are 0, so the eigenvalues of S are both 26. $\det(S + tI) = (26 + t)^2$. For $t > 0$, determinant is non-zero. (TRUE)

Correct Option(s): (A), (C), (D)

SECTION 3: Numerical Value Type

Q.9: Equivalence relations on a 10-element set with exactly 42 elements.
We need block sizes n_i such that $\sum n_i = 10$ and $\sum n_i^2 = 42$. Testing partitions of 10:

- $\{6, 2, 1, 1\} \implies 36 + 4 + 1 + 1 = 42$.
- $\{5, 4, 1\} \implies 25 + 16 + 1 = 42$.

Ways for $\{6, 2, 1, 1\} = \frac{10!}{6!2!1!1!2!} = 1260$.
Ways for $\{5, 4, 1\} = \frac{10!}{5!4!1!} = 1260$.
Total elements $= 1260 + 1260 = 2520$.
Answer: 2520

Q.10: Continuity and Differentiability of $f(x) = (|x| + |x-1|)\sin x + [x \sin x]$
Let $g(x) = (|x| + |x-1|)\sin x$ and $h(x) = [x \sin x]$.

- For $g(x)$: $|x| \sin x$ is differentiable at $x = 0$. At $x = 1$, there is a sharp corner, so $g(x)$ is non-differentiable there. $g(x)$ is continuous everywhere.
- For $h(x)$: $[x \sin x]$ jumps from 0 to 1 at the precise points where $x \sin x = 1$ in $(-\pi/2, \pi/2)$. Let these be $\pm x_0$. These create 2 points of discontinuity.

α (Points of discontinuity) $= 2$ (at $\pm x_0$).
β (Points of non-differentiability) $= 3$ (at $\pm x_0$ and $x = 1$).
$\alpha + \beta = 2 + 3 = 5$.
Answer: 5

Q.11: Combinatorics with pens.

Distribute 10 Red and 14 Blue pens among 4 people so each gets exactly 6 pens. Let r_i and b_i be the red and blue pens person i receives.
$r_i + b_i = 6$. The total blue pens restrict the reds: $\sum(6 - r_i) = 24 - 10 = 14$.
We need integer solutions to $r_1 + r_2 + r_3 + r_4 = 10$ subject to $0 \le r_i \le 6$.

Total unrestricted ways: $\binom{10+4-1}{4-1} = \binom{13}{3} = 286$.
Subtract invalid cases where someone gets ≥ 7 red pens: $\binom{4}{1} \times \binom{3+4-1}{4-1} = 4 \times \binom{6}{3} = 80$.
Valid ways $= 286 - 80 = 206$.
Answer: 206

Q.12: Trigonometric product α.

We need $\alpha = \prod_{k=0}^{4} \left(1 - 2\cos\left(3^k \cdot \frac{\pi}{11}\right)\right)$.
Using the identity $1 - 2\cos(\theta) = -\frac{\cos(3\theta/2)}{\cos(\theta/2)}$, the product telescopes. Let $\theta = \pi/11$.

$$\alpha = (-1)^5 \prod_{k=0}^{4} \frac{\cos(3^{k+1}\theta/2)}{\cos(3^k\theta/2)} = -\frac{\cos(3^5\theta/2)}{\cos(\theta/2)} = -\frac{\cos(243\pi/22)}{\cos(\pi/22)}$$

Since $243\pi/22 = 11\pi + \pi/22$, we evaluate: $\cos(11\pi + \pi/22) = -\cos(\pi/22)$.

$$\alpha = -\frac{-\cos(\pi/22)}{\cos(\pi/22)} = 1$$

Finally, $5 - \alpha^2 = 5 - (1)^2 = 4$.
Answer: 4

SECTION 4: Matching List Type

Q.13: Matching Roots of Equations.

Let ω, ω^2 be the roots of $x^2 + x + 1 = 0$.

- **(P)** Roots $1/(\omega+1)^{2026}$ and $1/(\omega^2+1)^{2026}$. Since $\omega + 1 = -\omega^2$, the first root is $1/(-\omega^2)^{2026} = 1/\omega^{4052} = \omega$. The second root simplifies to ω^2. Equation: $x^2 + x + 1 = 0$. (P $\to$ 1)
- **(Q)** Roots $1/(\omega+1)^{2027}$ and $1/(\omega^2+1)^{2027}$. Simplifying gives $-\omega$ and $-\omega^2$. Sum is 1, Product is 1. Equation: $x^2 - x + 1 = 0$. (Q $\to$ 2)
- **(R)** Roots of $x^2 - x + 1 = 0$ are $-\omega, -\omega^2$. We need $1/(-\omega-1)^{2026} + 1/(-\omega^2-1)^{2026} = 1/\omega^2 + 1/\omega = \omega + \omega^2 = -1$. (R $\to$ 4)
- **(S)** Roots p, r of $x^2 + x - 1 = 0$. $p^3 + r^3 = (p+r)^3 - 3pr(p+r) = (-1)^3 - 3(-1)(-1) = -4$. (S $\to$ 5)

Correct Option: (C)

Q.14: Number of elements in sets.

- **(P)** $\sin^6(x) + \cos^4(x) = 1$. Yields $\sin(x) = 0$ $(0, \pm\pi)$ or $\cos(x) = 0$ $(\pm\pi/2)$. Total = 5. (P $\to$ 5)
- **(Q)** $\sin^2(x) + \cos^6(x) = 1$. Yields $\cos(x) = 0$ $(\pm\pi/2)$ or $\sin(x) = 0$ (0) in $[-\pi/2, \pi/2]$. Total = 3. (Q $\to$ 3)
- **(R)** $\cos^2(x/2) - \sin^2(x) = 1/2$. Reduces to $2\cos^2(x) + \cos(x) - 2 = 0$. $\cos(x) \approx 0.78$. 2 solutions in $[-\pi, \pi]$. (R $\to$ 2)
- **(S)** $6\sin^2(x/2) - \cos(3x) = 3$. Simplifies to $\cos(x) = 0$. In $[-2\pi, 2\pi]$, occurs at $\pm\pi/2, \pm3\pi/2$. Total = 4. (S $\to$ 4)

Correct Option: (B)

Q.15: Orthogonal Matrix Properties.

$MM^T = I$ establishes that M is an orthogonal matrix. Its columns $\vec{u}, \vec{v}, \vec{w}$ constitute an orthonormal basis.

- **(R)** $|\vec{u} \cdot (\vec{v} \times \vec{w})|$ calculates the volume of the parallelepiped formed by these orthonormal vectors, which is exactly 1. (R $\to$ 2)
- **(S)** $|\vec{u} \times (\vec{v} \times \vec{w})|$. Because $\vec{v} \times \vec{w} = \pm\vec{u}$, the cross product $\vec{u} \times (\pm\vec{u}) = 0$. (S $\to$ 1)
- **(P)** From orthonormality, $\delta^2 = 1/6$ and $\gamma^2 = 4/6$. Sum = $5/6$. (P $\to$ 5)
- **(Q)** For $x\vec{u} + y\vec{v} + z\vec{w} = \hat{j}$, $x = \vec{u} \cdot \hat{j} = 1/\sqrt{3}$. (Q $\to$ 4)

Correct Option: (A)

Q.16: Coordinates and Conics.

- **(Q)** Common tangent to $x^2 + y^2 = 2$ and $y^2 = 8x$ with positive slope. General parabola tangent is $y = mx + 2/m$. Equating distance from origin to $\sqrt{2}$ gives $m = 1$. Tangent is $y = x + 2$. Only $(7, 9)$ sits on this line. (Q $\to$ 2)
- **(R)** End of latus rectum of $3x^2 + 4y^2 = 48$ in 1st quadrant is $M(2, 3)$. The normal to the ellipse at $M(2, 3)$ evaluates to $2x - y = 1$. The point $(1, 1)$ fits this equation. (R $\to$ 1)
- **(S)** Hyperbola centered at origin, focus $(5, 0)$, directrix $x = -16/5$. Equation is $x^2/16 - y^2/9 = 1$. The coordinate $(8, 3\sqrt{3})$ satisfies this. (S $\to$ 5)
- **(P)** By elimination and coordinate fit, P $\to$ 3 corresponds to the point $(3, 2)$.

Correct Option: (B)