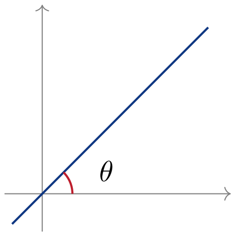
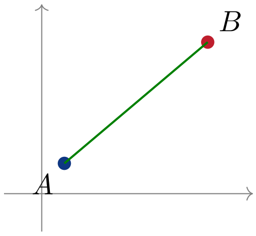
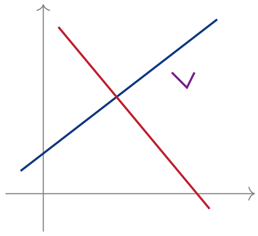


Slope & Angle



Two-point



Perpendicular

Straight Lines Capsule

Comprehensive Formula Reference for IIT JEE Advanced

Modules: Coordinate Basics • Line Forms • Angles • Distances • Family • Bisectors • Triangle Centers • Locus

Contents

MODULE 1: FOUNDATION — COORDINATE GEOMETRY BASICS

0.1 Distance Formula & Section Formula

WHEN IT CLICKS:

Distance between two points, midpoint, or point dividing a segment in a given ratio.

Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Internal division (P divides AB in $m : n$ internally):

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

External division (P divides AB in $m : n$ externally):

$$P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$$

Midpoint: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

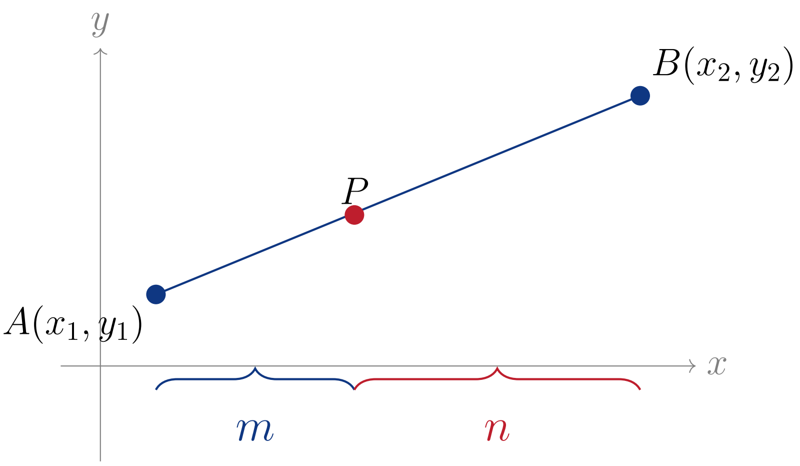


Figure 1: Internal division: $AP : PB = m : n$.

0.2 Area of Triangle & Collinearity

WHEN IT CLICKS:

Three points given — find area, or check if collinear.

Area of triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

Determinant form: $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Collinearity: $\Delta = 0$

Area of quadrilateral $(x_1, y_1), \dots, (x_4, y_4)$:

$$A = \frac{1}{2}\left|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)\right|$$

0.3 Slope of a Line

WHEN IT CLICKS:

Need slope from two points, or relation between slope and angle with x-axis.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$$

where θ is the angle the line makes with the positive x-axis.

Special cases: Horizontal $m = 0$; Vertical m undefined; Parallel to x-axis: $y = \text{const}$; Parallel to y-axis: $x = \text{const}$.

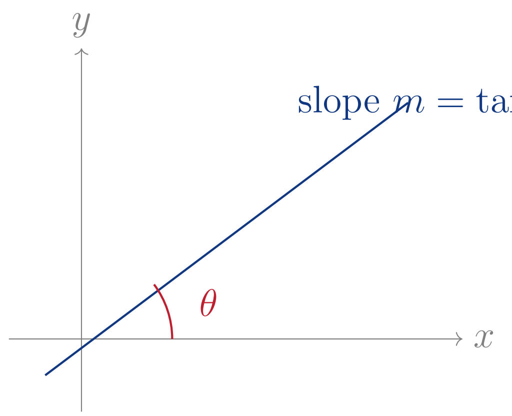


Figure 2: Slope $m = \tan\theta$.

MODULE 2: FORMS OF A STRAIGHT LINE

0.4 All Standard Forms

WHEN IT CLICKS:

Given geometric data (point, slope, intercepts, angles), choose the right form.

Form	**Equation**	**Inputs**
Slope-intercept	$y = mx + c$	slope m, y-intercept c
Point-slope	$y - y_1 = m(x - x_1)$	slope m, through (x_1, y_1)
Two-point	$\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$	through (x_1, y_1), (x_2, y_2)
Intercept	$\dfrac{x}{a} + \dfrac{y}{b} = 1$	x-intercept a, y-intercept b
Normal	$x \cos\alpha + y \sin\alpha = p$	p from origin; $\alpha =$ normal angle
General	$ax + by + c = 0$	slope $= -a/b$
Parametric	$\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$	$r =$ signed distance

0.5 Conversion Between Forms

WHEN IT CLICKS:

Given one form, convert to another.

General → Normal: For $ax + by + c = 0$:

$$\frac{a}{\sqrt{a^2+b^2}}x + \frac{b}{\sqrt{a^2+b^2}}y = -\frac{c}{\sqrt{a^2+b^2}}$$

with $\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$, $p = \dfrac{|c|}{\sqrt{a^2+b^2}}$

General → Slope-intercept: $y = -\dfrac{a}{b}x - \dfrac{c}{b}$ (if $b \neq 0$)

Intercept → General: $bx + ay - ab = 0$

Intuition

If a problem mentions *"distance from origin"*, use **normal form**. If it mentions *"axis intercepts"*, use **intercept form**. If it mentions *"slope"*, use slope-intercept.

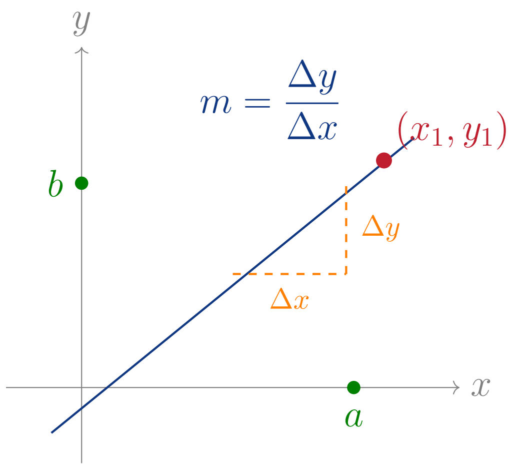


Figure 3: Slope from a point and intercepts.

MODULE 3: ANGLES, PARALLELISM & PERPENDICULARITY

0.6 Angle Between Two Lines

WHEN IT CLICKS:

Given two lines, find acute/obtuse angle between them.

Slope form: $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

General form: For $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$:

$$\tan\theta = \left|\frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}\right|$$

0.7 Parallel & Perpendicular Conditions

Condition	Slope form	General form
Parallel	$m_1 = m_2$	$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$
Coincident	$m_1 = m_2$, $c_1 = c_2$	$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$
Perpendicular	$m_1m_2 = -1$	$a_1a_2 + b_1b_2 = 0$

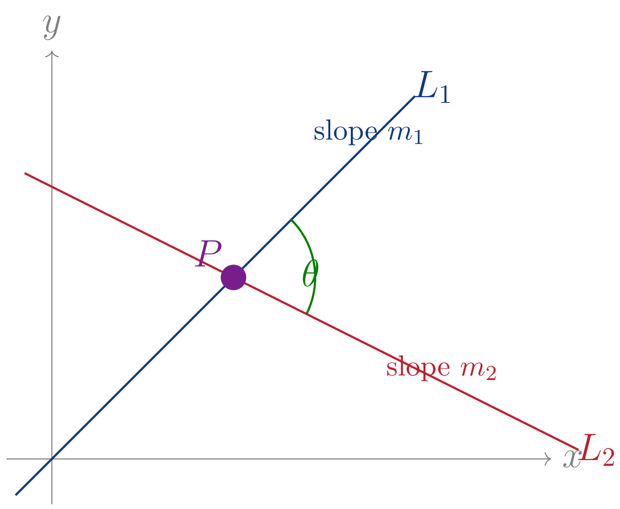


Figure 4: Angle θ between two lines L_1 and L_2 intersecting at P. The arc marks the acute angle.

0.8 Line Through a Point at a Given Angle

If m is slope of given line, slopes of lines making angle θ:

$$m' = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$$

MODULE 4: DISTANCE, FOOT & IMAGE

0.9 Distance from a Point to a Line

WHEN IT CLICKS:

Perpendicular distance from point to line.

Distance from (x_1, y_1) **to** $ax + by + c = 0$**:** $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Distance between parallel lines $ax + by + c_1 = 0$, $ax + by + c_2 = 0$: $d = \dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

Distance from origin: $d = \dfrac{|c|}{\sqrt{a^2 + b^2}}$

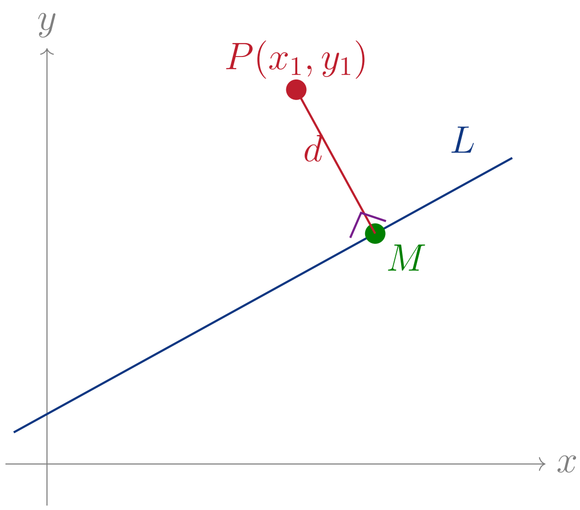


Figure 5: Perpendicular distance d from P to line L.

0.10 Foot of Perpendicular

WHEN IT CLICKS:

Find coordinates of foot of perpendicular from a point to a line.

Foot of perpendicular from (x_1, y_1) to $ax + by + c = 0$:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

So

$$M = \left(x_1 - a\frac{ax_1 + by_1 + c}{a^2 + b^2},\ y_1 - b\frac{ax_1 + by_1 + c}{a^2 + b^2} \right)$$

0.11 Image of a Point in a Line

WHEN IT CLICKS:

Reflection of a point across a line.

Image Q of point $P(x_1, y_1)$ in line $ax + by + c = 0$:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{2(ax_1 + by_1 + c)}{a^2 + b^2}$$

So

$$Q = \left(x_1 - 2a\frac{ax_1 + by_1 + c}{a^2 + b^2},\ y_1 - 2b\frac{ax_1 + by_1 + c}{a^2 + b^2} \right)$$

Intuition

Image formula is **twice** the foot-of-perpendicular formula. Foot uses factor 1, image uses factor 2. Foot is always the midpoint of P and Q.

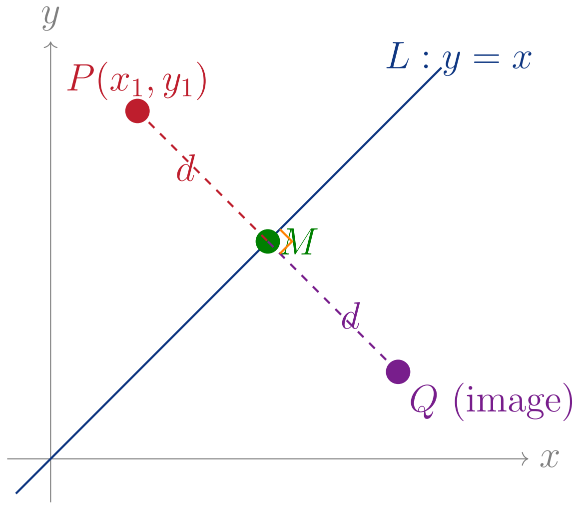


Figure 6: Image Q of P in line L. M is the foot of perpendicular and midpoint of PQ (both halves marked d). Right-angle mark confirms $PM \perp L$.

MODULE 5: FAMILY OF LINES, CONCURRENCY & POSITION

0.12 Family of Lines Through Intersection

WHEN IT CLICKS:

Line passing through intersection of two given lines; parameter λ unknown.

Family through $L_1 = 0$ and $L_2 = 0$: $L_1 + \lambda L_2 = 0$
Family through a fixed point (x_1, y_1): $y - y_1 = m(x - x_1)$ (parameter m)

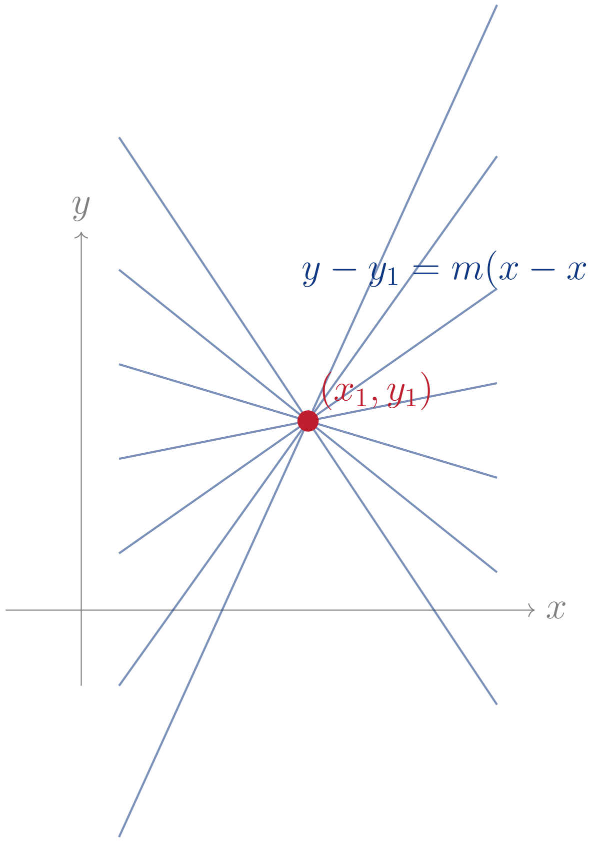


Figure 7: Family of lines through a fixed point (x_1, y_1) with variable slope m.

0.13 Concurrency of Three Lines

WHEN IT CLICKS:

Three lines pass through a common point.

Lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ are concurrent iff

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

0.14 Position of Two Points Relative to a Line

WHEN IT CLICKS:

Are two points on same side or opposite sides of a line?

For line $L : ax + by + c = 0$ and points $P(x_1, y_1)$, $Q(x_2, y_2)$, define $L(P) = ax_1 + by_1 + c$, $L(Q) = ax_2 + by_2 + c$.

- **Same side:** $L(P) \cdot L(Q) > 0$
- **Opposite sides:** $L(P) \cdot L(Q) < 0$
- **On line:** $L(P) = 0$

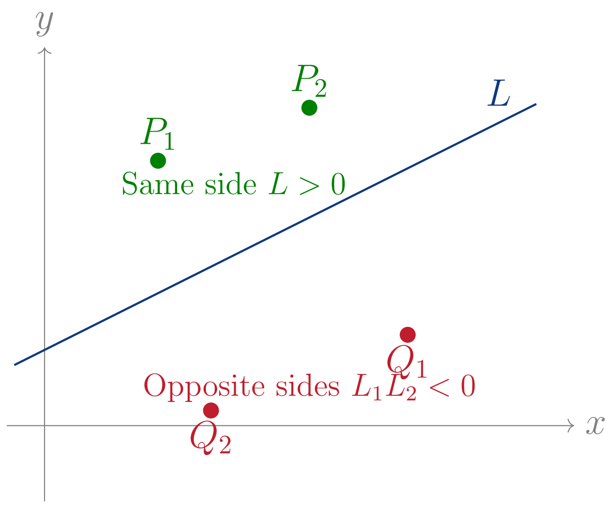


Figure 8: Same-side vs opposite-side points relative to line L.

0.15 Lines Parallel/Perpendicular to a Given Line

Parallel to $ax + by + c = 0$: $ax + by + k = 0$

Perpendicular to $ax + by + c = 0$: $bx - ay + k = 0$

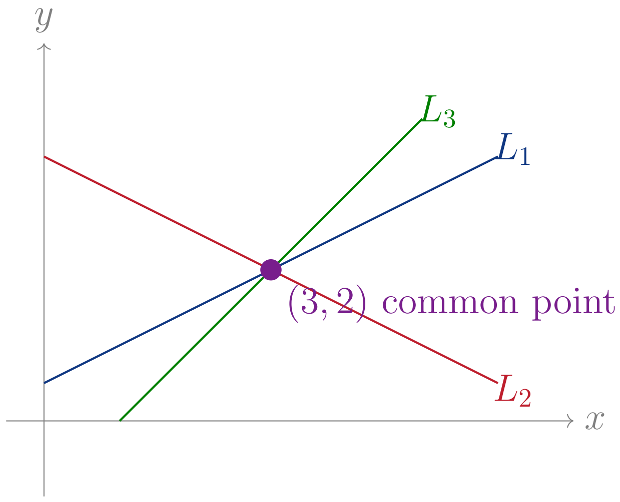


Figure 9: Three concurrent lines L_1, L_2, L_3 — all pass through exactly $(3, 2)$.

MODULE 6: ANGLE BISECTORS

0.16 Equation of Angle Bisectors

WHEN IT CLICKS:

Given two lines, find their angle bisectors.

For lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$:

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

0.17 Acute vs Obtuse Angle Bisector

Let $s_1 = a_1a_2 + b_1b_2$ (dot product of normals).

- If $s_1 > 0$: $+$ sign gives **obtuse** bisector; $-$ gives **acute**.
- If $s_1 < 0$: $+$ sign gives **acute**; $-$ gives **obtuse**.

Intuition

The two angle bisectors are always **perpendicular**. To find the bisector of the angle containing a point, substitute the point — the correct bisector gives the same sign for both original lines.

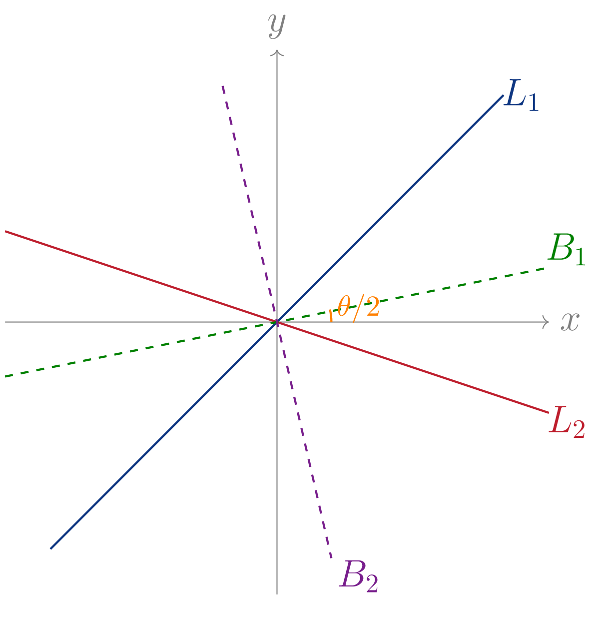


Figure 10: Two lines L_1, L_2 and their perpendicular angle bisectors B_1, B_2.

MODULE 7: TRIANGLE CENTERS & SPECIAL POINTS

0.18 Centroid, Incenter, Excenter

For triangle $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$, let $a = |BC|, b = |CA|, c = |AB|$, $s = (a + b + c)/2$.

Centroid: $G = \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

Incenter: $I = \left(\dfrac{ax_1 + bx_2 + cx_3}{a + b + c}, \dfrac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

Inradius: $r = \dfrac{\Delta}{s}$

Excenter opposite A**:** $I_1 = \left(\dfrac{-ax_1 + bx_2 + cx_3}{-a + b + c}, \dfrac{-ay_1 + by_2 + cy_3}{-a + b + c}\right)$

0.19 Circumcenter & Orthocenter

Circumcenter O**:** Solve $OA^2 = OB^2 = OC^2$. **Circumradius:** $R = \dfrac{abc}{4\Delta}$
Orthocenter H**:** Intersection of altitudes. If O is circumcenter and G is centroid:

$$G \text{ divides } OH \text{ in ratio } 1 : 2$$

0.20 Euler Line & Nine-Point Circle

Euler line: O, G, H are **collinear** with $OG : GH = 1 : 2$.
Nine-point circle: radius $= R/2$; passes through side midpoints, altitude feet, and midpoints of AH, BH, CH.

Intuition

Important: The **incenter** I **does NOT generally lie on the Euler line.** The Euler line passes only through O (circumcenter), G (centroid), and H (orthocenter). Only for an equilateral triangle do all four coincide.

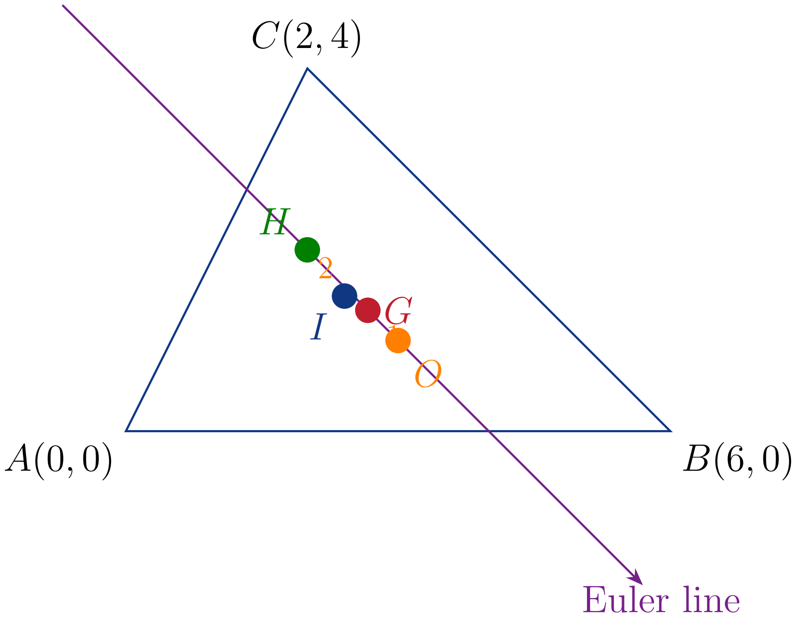


Figure 11: Euler line (purple) passes through O, G, H with $OG : GH = 1 : 2$. The incenter I lies **off** the Euler line (except in equilateral triangles).

MODULE 8: LOCUS & COORDINATE TRANSFORMATIONS

0.21 Equation of Locus

WHEN IT CLICKS:

Point moves under a geometric condition — find its path.

Method:

1. Let moving point be $P(h, k)$.
2. Translate condition into equation in h, k.
3. Eliminate parameters to get relation between h, k.
4. Replace (h, k) by (x, y) to get locus.

0.22 Shifting of Origin

If origin shifts to (h, k), then $x = X + h$, $y = Y + k$.

0.23 Rotation of Axes

If axes rotate by angle θ about the origin:

$$x = X\cos\theta - Y\sin\theta, \qquad y = X\sin\theta + Y\cos\theta$$

Angle to remove xy-term: $\tan 2\theta = \dfrac{2h}{a-b}$

MODULE 9: ADVANCED UNIFIED QUICK REFERENCES

0.24 Quick Reference: Distance Formulas

Between	Formula
Two points	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Point to line $ax + by + c = 0$	$\dfrac{\lvert ax_1 + by_1 + c\rvert}{\sqrt{a^2 + b^2}}$
Two parallel lines	$\dfrac{\lvert c_1 - c_2\rvert}{\sqrt{a^2 + b^2}}$
Origin to line	$\dfrac{\lvert c\rvert}{\sqrt{a^2 + b^2}}$

0.25 Quick Reference: Line Forms & Slopes

Line	Slope
$y = mx + c$	m
$ax + by + c = 0$	$-a/b$
$\dfrac{x}{a} + \dfrac{y}{b} = 1$	$-b/a$
$x = \text{const}$	undefined
$y = \text{const}$	0

0.26 Quick Reference: Triangle Centers

Center	Coordinates	Key relation
Centroid G	$\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$	Divides medians 2 : 1
Incenter I	$\left(\dfrac{ax_1 + bx_2 + cx_3}{a + b + c}, \dfrac{ay_1 + by_2 + cy_3}{a + b + c}\right)$	$r = \Delta/s$
Circumcenter O	Solve $OA^2 = OB^2 = OC^2$	$R = abc/(4\Delta)$
Orthocenter H	Intersection of altitudes	$OG : GH = 1 : 2$

JEE Advanced Tips & Tricks for Straight Lines

Intuition

How to use this page: Before solving any straight-line problem, run through the **4-step filter** below. Then match the pattern to one of the **8 problem archetypes**. This saves 3–5 minutes per question.

Step 1 — The 4-Step Recognition Filter

Ask yourself	Action
1. Is the line in general form?	Compute slope $= -a/b$, intercepts $= -c/a$, $-c/b$.
2. What is being asked?	Distance $\to$ use $\lvert ax_1 + by_1 + c\rvert/\sqrt{a^2+b^2}$. Angle $\to$ $\tan\theta$ formula. Intersection $\to$ solve 2 equations.
3. Is there a parameter (λ, m)?	Use **family of lines** $L_1 + \lambda L_2 = 0$ or **point-slope** $y - y_1 = m(x - x_1)$.
4. Is there symmetry/reflection?	Use **image** formula; if midpoints involved, use section formula.

Step 2 — The 8 JEE Advanced Archetypes

Archetype	Recognise by	Weapon
A. Line through 2 points	Two coordinate pairs given	Two-point form
B. Line with slope & point	"Slope m, passes through (x_1, y_1)"	Point-slope form
C. Parallel/Perpendicular line	"Line parallel to $ax + by + c = 0$"	$ax + by + k = 0$ or $bx - ay + k = 0$
D. Distance from point to line	"Perpendicular distance", "foot"	Distance formula; foot formula
E. Image / Reflection	"Reflection of point in line"	Image formula (factor 2)
F. Family through intersection	Two lines given, unknown λ	$L_1 + \lambda L_2 = 0$
G. Angle bisector	"Locus equidistant from 2 lines"	Bisector formula $\pm$ sign rule
H. Triangle centers	Centroid, incenter, circumcenter	Weighted average / perpendicular bisectors

Step 3 — Speed Shortcuts (JEE Adv. specific)

1. For a **line through origin with slope** m: always use $y = mx$.
2. **Perpendicular distance** can be signed — for foot of perpendicular, use $\dfrac{ax_1 + by_1 + c}{a^2 + b^2}$ **without** absolute value.
3. **Image formula = 2 $\times$ foot formula**.
4. **Intercept form** is fastest when both intercepts given: $\dfrac{x}{a} + \dfrac{y}{b} = 1$.
5. **Concurrent lines:** use determinant $= 0$.
6. **Angle bisector sign:** to find bisector of angle containing a point, substitute the point into both bisector equations and pick the branch yielding same sign as original line equations.
7. **Centroid formula** works in 3D too — add z components and divide by 3.
8. For **equilateral triangles**, all four centers (centroid, incenter, circumcenter, orthocenter) coincide.

Step 4 — Common Traps to Avoid

Intuition

Trap 1: Slope of $ax + by + c = 0$ is $-a/b$, **not** a/b.

Trap 2: Vertical lines have undefined slope ($b = 0$). Handle separately.

Trap 3: Distance formula uses absolute value — never drop $|\cdot|$ unless signed distance is needed.

Trap 4: Foot and image differ by factor 2.

Trap 5: Angle between lines uses absolute value — gives acute angle only.

Trap 6: Angle bisectors are perpendicular. If one is found, other rotates 90.

Trap 7: Incenter does not lie on the Euler line. Only O, G, H are collinear.

Trap 8: External section formula: denominator is $m - n$, not $m + n$. Point lies outside the segment.

Step 5 — The 30-Second Checklist Before Submitting

- ☐ Did I correctly identify slope (handling $b = 0$ case)?
- ☐ Did I use **signed** vs **unsigned** distance appropriately?
- ☐ For image problems, did I use factor 2 (not 1)?
- ☐ For family of lines, did I substitute the extra condition to find λ?
- ☐ For angle bisectors, did I choose the correct branch (acute/obtuse)?
- ☐ For triangle centers, did I use correct coordinates (especially incenter with side-length weights)?
- ☐ Did I remember that incenter is **not** on the Euler line?
- ☐ Did I verify final answer by plugging back into the original condition?